

## **Learning Math: Basic concepts, math difficulties, and suggestions for intervention**

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*Math difficulties share many common features with reading difficulties. In as much as they do so, the general approach to reading disability overlaps with math disability. Both math and learning to read share several domain-general features such as long-term and short-term memory, successive and simultaneous processing, flexibility in strategies, attention to details and knowledge-base. The paper reviews the foundational concepts for learning math -- magnitude, value, procedure, and estimation. Math disability, therefore, can be identified in terms of the domain-general cognitive processes as well as difficulties in mathematical operations involving Magnitude, Value, Procedure, and Estimation. The review, in its final section, presents some empirical studies of math learning and cognitive intervention from contemporary literature. The authors conclude that there are many different kinds of learning difficulties, and many varieties of math learning disabilities. It should be recognized that many areas of the brain contribute to the successful learning and application of math skills.*

### **Introduction**

Math difficulties share many common features with reading difficulties. In as much as they do so, the general approach to reading disability overlaps with math disability.

Where do we begin to show the shared and non-shared features between the two disabilities? Let us begin with distinguishing between general and specific features and abilities in reading and math, in other words, between "domain-general" and "domain-specific" features and abilities involved in competent performance in math and reading. Some suggestions from researchers (Busse, Berninger, Smith, & Hildebrand 2001; Geary, 2004), who have knowledge of both disabilities, are considered. We have attempted to cast their views within the PASS theory framework (Das, Naglieri, & Kirby, 1994).

Domain-general features include short-term memory for sequences. This can be recast as a part of *Successive Processing*. Successive processing is a mental process by which the person integrates stimuli in a specific serial order. Stimuli form a chain-like progression.

Successive processing is not necessarily verbal; nonverbal sequences as in dance or imitating a series of hand movements as well as continuation of a repetitive series such as writing a string of ++\_++\_++\_++\_+ or 1121121121...requires successive processing.

Several strategies for intervention in math disability commonly recommended are chunking information into bite-size units rather than carrying in memory repetition of a long series of number sequences: 112112112...can be chunked as 112/112/ etc. Note-taking while doing long division and multiplication, or listening to a teacher's long statement, reduces the demand on carrying information in the head, decreasing the load on working memory as a consequence.

Long-term memory is essentially packed with stored knowledge. Both reading and math must draw upon this knowledge-base. Without prerequisite knowledge of numbers and basic operations, math problems cannot be solved. Comprehension of a math problem also vitally depends on the same processes as comprehension of materials that a child reads.

Examples of simultaneous processing include seeing patterns and configurations in geometry and transpositions in simple equations. In higher grades, even staples of math curricula, such as arithmetic progressions, simultaneous equations, and permutations and combinations are closely linked to the use of simultaneous processing strategies! Seeing similarities between two problems and transferring procedures learned for one to the other are often missed by children with math disability. Although step-by-step procedures in solving a problem are perhaps closer to successive processing and planning, seeing similarities is a basic categorization process included in the class of simultaneous processing. We cannot exaggerate the role of simultaneous processing in learning math.

Processing strategies are obviously important for competent performance in both reading and math. Specific strategies for math

undergo changes as the material demands flexibility. Shift from counting with fingers to conceptualizing basic operations in mental arithmetic occurs, a sign of maturity in the child learning math. Similarly, strategies for estimating an answer in long addition, subtraction, multiplication and division, used when the answer cannot be accurately given (Siegler & Booth, 2004), may appear to be specific for math; however, in coping with reading short and long words and comprehending the meaning of a word from its context involve processes shared with math. Change in strategies, or flexibility in strategy use is a central requirement of *Planning*. Good planning is almost synonymous with flexibility, lack of rigidity, heeding feedback and correcting one's approach to solution.

As we write more about these domain-general processes, the overarching value of planning in the completion of more advanced math tasks will become apparent.

#### *Foundational Concepts*

What are the two foundational concepts for learning math? Let us consider dealing with numbers and objects. Both have magnitude (size) and value. Even an infant can make out a large object from a small object (size). At a slightly higher stage of development, children can arrange 3 to 4 objects from big to small, or differentiate a densely distributed field of dots from a sparsely distributed field. At that stage, the concept of more and less may not have been clearly developed.

Example — 5 candies arranged in a long versus a short row — which one does the child choose? The long one; because “longer is more.”

Size and weight are similarly confounded—the bigger, the heavier. Piaget's conservation tests for mass and volume are good devices for teaching the disconnection between size and volume, length and numerosity.

Seriation is also a basic math requirement. However, is it simultaneous? Is it Successive? Early research has shown that conservation and other concrete operation tasks are essentially examples of simultaneous processing (Das, Naglieri, & Kirby, 1994).

*How to Help the Development of Math Abilities*

*Some theoretical assumptions.* Learning math is concerned with two basic concepts: *magnitude and value*. Math also demands two processes; one is the *procedure*, which is a step-by-step thinking out of an arithmetic problem (e.g., divide 26 by 3) and *conceptualizing or comprehending* the problem. We will explain each one.

Besides the basics of magnitude and value, and the two processes that help math, *working memory* (WM) is a major cognitive process that is essential for math/arithmetic. Those who are weak in WM need to be assisted. How?

One of the procedures for boosting WM is to reduce the to-be-remembered material into smaller bits (chunks) and then remember them. The other procedure to lighten the burden on WM is to use notes or noting down the calculations as children proceed step-by-step when solving a division or multiplication (e.g., carry over).

*More on magnitude, value, procedure and conceptualization.* Magnitude is size, it is bulk, it is quantity: Big, medium, little, large and small. Each of these indicates magnitude: A big object is larger than a small object. Even an infant has a sense of magnitude; knows that a big object can hide a small one behind it. Probably this is a module, a modular concept present at birth, or at least a blue-print exists to start with and it is fed by experience.

Value refers to the value of a number: The size of the number does not necessarily correspond with its value. A number may be of the same size as another (e.g., 29 & 45), but one of them is of smaller value than the other. Value is a concept that has to be learnt, it is not innate nor is it independent of the general level of the cognitive development of the child.

The difference between magnitude and value is analogous to the distinction between speaking and reading. While speaking occurs naturally and perhaps a blue-print exists in human brain for speaking, reading has to be learnt, reading is acquired.

The question then is asked -- In tracing the difficulty of a child who is beginning to do math, should we first suspect that the difficulty is related to learning value and not magnitude? Perhaps so.

The other aspect of math learning is related to procedure and conceptualization of the problem: Within the step-by-step procedure for working out a math problem, memory is obviously important along with the logical sequence of the calculation. Both help the individual to plan a course of action, and guide his actions. Planning processes including strategies may be as important as knowledge of the procedure and memory.

Let us reflect a bit on memory, of which there are several kinds—procedural-declarative, short term-long term, episodic-semantic, explicit-implicit. Can this kind of memory be named as procedural memory? It is tempting to name it as such; a memory of procedure is procedural memory! However, procedures have to be learned and over-learned, have to go through deliberation and thinking many times to become automatic. Thus it passes through an explicit learning stage to become implicit, not requiring conscious effort.

Short-term memory (STM) and long-term memory (LTM) are required for successful working out of a step-by-step procedure; this is our suggestion. For example, while doing a division (e.g., 26 divided by 3), the procedure involves both kinds of memory: I can keep in STM  $26 - 24 = 2$ , and try to divide 2 by 3. I also use my long-term rote memory when multiplying 3 by 8 (i.e., multiplication table).

How can we detect where the individual child's difficulties may lie within the operation of the procedure? Is it in STM? Is STM unusually short for his/her age? In this case, smaller chunks of the steps can be recommended. Break the steps down into bite-size chunks to fit the individual's capacity. Is it in rote memory, such as multiplication tables or in basic facts which should have been accessed without effort as fast and automatic (e.g., adding/subtracting simple one-digit numbers) procedures? Yes, probably so.

Recent researchers tend not only to view problem-solving (conceptual) separately from procedural operation, but also regard the two to have distinct locations in the cortex (Parietal - Occipital, and Frontal-Temporal

respectively). When active planning is demanded in step-by-step operations rather than accessing or reaching into long-term memory, then, of course, the frontal lobes must be involved.

The above discussions should guide the teacher and the clinician to diagnose math difficulty and to formulate a remediation programme.

*Math Disability: Difficulties with Magnitude, Value, Procedure, and Estimation*

We now have a fairly good idea of the importance of the first three in programmes for helping children to learn math. The fourth is *estimation*, the last but not any less important than the other three. Good teaching of math, some say, should begin with teaching children, not the exact answer to a simple math question, but with teaching how to estimate--the nearest approximation to the correct answer. You may ask a child in Grade 2: If we add 69 to 79, would the total be more like 89 or more than 100? Estimations not involving numbers can be begun even earlier, in kindergarten classes: Between a car, a bus and a train, which is the longest. Or, if you put two cars end to end, would it be longer than a train? Estimates like this do not really require number concepts, whereas the above example of addition do require it to a certain degree.

*Links to planning.*

How we estimate is linked to the PASS process, especially to planning. "What's the distance of the moon from the earth?"—we asked a grad student from India who knows the height of Mount Everest from his Grade 3 books. But he was known to give an answer right away without even thinking! He replied 5 miles? The questioner, who was really intent on teasing him, said, "But Mount Everest is higher than 5 miles; so the moon would be getting torn apart every time it passes over Mount Everest!"

*Number-line helps estimation.*

A psychology professor, Siegler (1988), has made a life-long study of how children learn mathematics and, in fact, asks the question—what develops in development? In development of math learning, for

example, a sense of estimation develops. It is linked to an internal representation; a handy tool for this is the number-line.

American schools teach using a number-line; if a number is to the left of a number on the number-line, it is less than the other number. If it is to the right then it is greater than that number. Siegler found that young children from kindergarten to Grade 2 can use a number-line from zero to 100. Numbers closer to the starting point of zero are estimated better than those closer to 100. For children in this age range, the distance between two or three big numbers closer to 100 is estimated to be shorter on the number-line, whereas between smaller numbers, estimation is more accurate. The same inaccurate estimation is not seen for older children for numbers between 0 and 100. However, when the number-line stretches to 1000, the older children pass through the same stage of estimating distances between numbers; the numbers much farther from zero, nearer to 1000 are estimated to have a shorter distance than numbers between 0 and 100.

*Developing automaticity.*

Why may some children do even simple addition and multiplication faster than others in the same class? A major reason is how familiar the children are with numbers—can they recognize the numbers faster? Almost automatically without effort? Similar to fast readers who begin to recognize and say letters of the alphabet earlier than slow readers, familiarity with the number helps children in their estimation like familiarity with a word—how early a word or number was learnt and used speeds up naming time for words or numbers no matter how long these may be and how difficult it may be to read them. Your 7-digit telephone number is more easily recognized than any string of 7 numbers!

*More on How Children Learn Math*

Research contributions made in the mid-1980s by psychologists such as Siegler, interested in children's mathematics performance, have helped to give direction to later research. Siegler and his associates (Siegler, 1988; Siegler, & Booth, 2004; Siegler, & Opfer, 2003) have developed a model to explain one difference between students who are high math achievers and those who are low achievers. He suggested that the

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strategies that students use to answer arithmetic problems were responsible for part of the success or failure that resulted from their use. The level of understanding a child has, relating to the type of math question asked, determines the strategy likely chosen to assist in its solution. Consider addition, for example. There are four general types of strategies used by children when solving basic addition problems: visibly counting fingers, representing numbers with fingers and adding without visibly counting, verbal counting, and finally retrieval of an answer from long-term memory.

Following Siegler's Strategy Choice Model, Geary and his associates (Geary, 1994; Geary, Hoard, & Hamson, 1999) conducted several studies. An example from their research is a paper by Geary and Brown (1991), two authors who have interest in both the areas of learning disabilities and cognitive abilities that promote math competency (Geary, 1994). Their paper, which we discuss below, was one of the first articles that directly investigated the area of math disability. It makes sense that once an area of learning is found to be unusually difficult for only some students, a comparison must be made to help understand the relative differences between the groups. Results from this study found significant differences between each group in the types of strategies used, effectiveness of strategy chosen, and level of math competency. A variety of strategies are used by students even at the same level of development, and by the same student at different times. Self-talk or verbalization of strategies that a student uses to solve an arithmetic problem, therefore, helps in choosing the strategies best suited for that individual student. Verbalization also facilitates shifts in the strategies when these are necessary for different types of problems, as Naglieri and his colleagues found in their research, which will be discussed later.

#### *Conclusions from Research on Math Competence*

We conclude from research in math competency that higher-level math achievement requires multiple cognitive processes, including quantitative knowledge, quantitative reasoning, short-term and working memory, visual processing, and processing speed. Research suggests that, since a multidimensional set of traits contribute to the effects of having a math disorder, tools or measures used to predict the future development of a math disorder must also be multidimensional. The PASS theory appears to be quite useful as a tool in aiding the



development of both assessment and remediation procedures that use a multidimensional approach.

*PASS theory explanation of math performance: Selected studies.*

Kroesbergen, Johannes, Luit, and Naglieri have addressed this concern in a study conducted in 2003. The study examined the relationships between mathematical learning difficulties (Math LD) and the *Planning, Attention, Simultaneous, Successive* (PASS) theory of cognitive processing. The measurement tool that was developed from the PASS theory was used in this study. The authors say that the development of other approaches to intelligence testing, such as the *Kaufman Assessment Battery for Children* (K-ABC; Kaufman & Kaufman, 1983) and the *Cognitive Assessment System* (CAS; Naglieri, & Das, 1997), is of obvious relevance for both diagnostic and instructional purposes. Because these theory-based tests measure ability as a multidimensional concept, they may provide more information on specific components and processes than a test designed to measure general intelligence, such as the WISC-III. Since the CAS is based on the PASS theory, its results have a more significant purpose than description alone. Treatment based on the PASS theory can be developed to address the specific needs of each test-taker.

The study posed two questions: Do students with Math LD exhibit different PASS cognitive profiles than their typically achieving peers? What is the relationship between cognition and improvement in mathematics achievement? The participants recruited for this study included 137 students with Math LD, scoring below the 25<sup>th</sup> percentile in a criterion-referenced math test, and 185 students without specific learning difficulties enrolled in general education elementary schools, and 130 students with Math LD enrolled in special education elementary schools. The study was conducted in the Netherlands. Pretest results for each child were gathered using the CAS. Part of the Mathematics Strategy Training for Educational Remediation (MASTER) was used as the instructional program for the students, which places its emphasis on the encouragement of strategy use when answering multiplication problems. During the 30 intervention lessons, strategy use and automated mastery were emphasized. Multiplication tests were administered before and after the intervention period and consisted of a basic skills test, an automaticity test, and a word-problems test. All three tests belong to the intervention program used in the study.

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Comparing the results of the CAS to the U.S. norms, the researchers found a significant difference in 5 out of the 12 CAS subtests. The differences, though significant, were all less than one standard deviation (3 points) from the mean. Multivariate analyses of variance showed that the students with Math LD performed significantly lower than their normal achievement peer group on all of the PASS scales ( $p = <0.001$ ). Students with Math LD in the special education classes also scored significantly lower than their Math LD peers in regular classrooms ( $p = <0.001$ ). The results from all the Math LD students allowed the researchers to identify three different types of math difficulties; students with difficulties in learning basic multiplication facts, students with difficulties in reaching automated mastery of basic facts, and students with difficulties in learning to solve math word-problems but no difficulties with basic multiplication facts. Of the 267 students with Math LD, 45 were found to clearly fit into one of the three groups. Within-group analysis of variance showed patterns for the three types of math difficulty found; students with difficulties learning multiplication skills scored low on all four PASS profiles, students with automaticity problems produced low scores on Planning, Attention, and Successive scales and relatively high scores on the Simultaneous scale, and students with difficulties solving math word-problems produced relatively lower scores on the Successive scale and relatively high scores on the Simultaneous scale.

The next part of the study focused on the relationship between an improvement in a student's math performance and the student's CAS scores. The comparison was made between the effect of a math intervention on students with Math LD and those with no specific cognitive weakness. Results from this comparison showed no significant differences in achievement between pre- and posttest for either group of students. An earlier study found that an intervention program focused on improving planning in students with a cognitive weakness in that area showed more improvement as a function of the planning intervention than in students without a cognitive weakness in the area of planning (Naglieri & Johnson, 2000).

A suggestion was made in the article about the reason there were no significant improvements in CAS scores after the intervention. Since the current intervention was not directly aimed at increasing a specific

PASS-theory cognitive dimension, there was a smaller effect of the intervention on CAS scores. Though the relationships found in this study were not very strong, there were important characteristics highlighted in the cognitive ability of students with Math LD that will contribute to the current level of knowledge in this area. According to this study, the key in successful resolution of math word-problems seems to lie in the cognitive areas of Attention and Successive processing.

Just around the same time as Naglieri (2003), another group of researchers were looking into the assumptions being made by the community of researchers and scholars in the area of Math LD. Aunola, Leskinen, Lerkkanen, and Nurmi (2004) were working on a study that could add a new dimension in the description and prediction of Math LD. The authors found that the most important component relating to the prediction of Math LD was the relationship between early measures of cognitive development and mathematic performance, to the outcome of later mathematic performance and the diagnosis of Math LD. Their study focused on examining both the developmental trajectories of overall math performance and the heterogeneity in math development trajectories during the period when children transferred from preschool to grade 2. A few researchers have already suggested that it may be necessary to identify heterogeneity in individuals' developmental trajectories rather than measuring the development purely on a sampling level (Bergman, Magnusson, & El-Khoury, 2003; Muthur & Muthur, 2000; Nagin, 1999). It is naive to assume that all students progress in nearly equal ways, or that all students with Math LD would acquire math competence at the same rate. Listening comprehension, cognitive, metacognitive, and attentional antecedents are suggested to play a major role in the development of math performance from preschool to grade 2. Four main questions lead the investigation: How does children's math performance develop from preschool to grade 2? What are the major cognitive antecedents of math performance during this period? To what extent do boys and girls differ in the level and growth of their math performance? What kinds of trajectories can be identified in the development of math performance by applying a person-oriented approach; that is, to what extent is there heterogeneity in the growth trajectories of math performance, and what are the major antecedents of math development in each trajectory?

The participants in this study were 194 children, 5 - 6 years old and in the transition between preschool to grade 1. They were a homogeneous group, however three children skipped grade 1, seven children had special education curriculum subjects taught to them in the regular class, and four students were placed into special education classrooms. Preliminary analysis showed that initial difference in the students' math performance was related to the parental level of socioeconomic status (SES), however, parents with different SES backgrounds did not differ in the amount of arithmetic they had taught their children. Because the authors wanted to examine the developmental dynamics of math performance across six measurement points using latent growth curve modeling, they had to create a set of equal measures across all six measurement points and ensure that the results were not influenced by either floor or ceiling effects. Mathematic performance was measured using items addressing *Knowledge of ordinal numbers*, *Knowledge of cardinal numbers*, *Number identification*, *Word problems*, and *Basic arithmetic*. Cognitive antecedents were measured using items addressing *Counting ability*, *Counting forward*, *Counting backward*, *Counting by number*, *Visual attention*, *Metacognitive knowledge*, and *Listening comprehension*. Specific items were pulled from currently-used measures with proven validity and split-half reliability.

Results showed a high stability in math performance across the six measurements, and the variance of the latent math performance constructs increased throughout the measurements. The results also suggest that the development of math performance across all measurements shows a cumulative pattern: The higher the initial level of math performance, the faster its rate of growth from preschool to grade 2. Boys also showed a faster rate of growth than girls.

Finally, the researchers set out to identify the various learning trajectories that could be related to any particular group of students. To identify different developmental trajectories, they performed statistical procedures using mixture models with different numbers of latent classes. Trajectory classes are then formed on the basis of the means of level and slope. The number of latent classes were chosen by considering three different criteria; the fit of the model, classification quality, and the usefulness and interpretativeness of the latent classes in practice.

Results clearly supported a two-class solution. A three-class solution would also fit the data well but, as the authors noted, there would only be 13 students in the third group. The first group was made up of 73 students classified as *high performers*, and 121 children classified as *low performers*. The variance in the level and growth rate of each group was statistically significant. These results confirm the findings from earlier in the study. Cognitive predictors, counting ability, visual attention, metacognitive knowledge, listening comprehension, and sex, were added to predict children's group membership. Results showed that two cognitive antecedents, plus gender, predicted the children's class membership; Higher levels in the student's counting ability ( $p < 0.01$ ) and visual attention ( $p < 0.05$ ) in the first measurement made it more likely that those students would be included in the high performers group. Being a boy also increased the likelihood of membership in the high performers group ( $p < 0.05$ ).

### Concluding Remarks

The past 30 years have been witness to a great many discoveries in the area of math learning disability. Our currently accepted truths were, not so long ago, merely thoughts created by scholars, who either built upon prior knowledge, or were able to see something in a way no one else had been able to do. It is now understood that there are many different kinds of learning difficulties, and there are likely many varieties of math learning disabilities. We know that many areas of the brain contribute to the successful learning and application of math skills. There are patterns in tests that measure cognitive ability, that can predict not only Math LD in general, but specific types of problems that can affect math performance. We now know that students who begin school with a solid understanding of basic math concepts are likely to achieve greater levels of math competency in their lives, and those who have difficulty with math in the early years will generally struggle with math for most of their school careers.

Researchers who have chosen to devote their efforts to the research of mathematic abilities and disabilities are few in number but they are playing an incredible role in setting the foundation for future researchers. Siegler, Mazzocco, and Brown, are only a few who have, over the past 30 years, continued to contribute to the level of understanding and knowledge we currently have. Though they will be

able to accomplish a small portion of what they have set out to do, their work will inspire, and give direction to others who will perpetuate the discovery of knowledge. It is not the status quo that pushes us to acquire knowledge, but the acceptance that our current level of knowledge is not only finite, but also full of incredible potential.

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